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梁索耦合结构的非线性振动

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摘要：在一组能描述梁索耦合结构中主缆曲率和吊索变形对系统影响的偏微分方程组基础上，通过 Galerkin 方法得到了系统在时域上一次截断的非线性常微分方程组．用多尺度法分析了所得的非线性常微分方程组．得到了主共振和 1∶2 内共振情况下以作用在梁上荷载的幅值为参数的振幅响应曲线和一次近似解析解．结果显示，一次近似解析解有良好的精度．系统发生内共振时，随激励幅值变化存在振幅突然变化的跳跃现象，而且低频共振时发生跳跃的分岔值小于高频共振时的分岔值．这说明低频共振更容易使结构发生大幅振动．该结果对工程应用有一定指导意义．

关键词：梁索耦合结构；非线性振动；Galerkin 法；多尺度法；1∶2 内共振
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Nonlinear Vibration of Coupled Structure of Cable-stayed Beam

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Abstract：Based on a series of partial differential equations describing the effect of curvature of the main cables and deformation of the stay cables on the structure，the non-linear ordinary differential equations of the first order approximation on time-domain are obtained by Galerkin method and studied by multi-scale method. An analysis is made of the principal resonance and 1 ∶ 2 internal resonance. The analytical solutions of the first order approximation and the vibration amplitude response curves for the amplitude of excitation as a parameter are derived. The results indicate the first order approximate analytical solutions have a higher accuracy. In the internal resonance condition，there are jump phenomena of the vibration amplitudes of the main cables and the beam. But the low-order resonance bifurcation values of the jump phenomena are less than the high-order resonance bifurcation values，which indicates that the low-frequency resonance vibration is easier to generate substantial vibrations than the low-frequency. The study results are significant for engineering applications

Key words：coupling of cable-stayed beam；non-linear vibration；Galerkin method；multi-scale method；1 ∶ 2 internal resonance

梁索耦合结构是现代大型结构普遍采用的结构类型．例如大跨度桥梁和悬索承重屋盖．一般情况下该结构类型是强非线性结构，对其进行研究有较大的难度．例如悬索桥的振动特性一直是桥梁工程界面临的重大挑战．自 1940 年，美国 Tacoma 桥在风致振动中倒塌，以及 1953 年瑞典 villars 车站大厅悬索屋盖在风致振动中破坏后，有很多学者对梁索耦合结构的动力学特性进行了大量研究并建立了不同的数学模型[1]．其中 A. C. Lazer 等[2]建立了一组考虑悬索桥吊索松弛对系统影响的数学模型，并对该模型进行了研究．但该数学模型中没有反映主缆曲率对系统的影响．之后 Gabriela 等[3]证明了 A. C. Lazer 和 P. J. Mckenna 建立的单侧约束下梁索耦合微分方程组解的存在惟一性．N. U. Ahmed 等[4]用数值方法研究了文献[2]中的偏微分方程组．R. H. Plaut[5]研究了主缆与加径梁间的对角连接松弛对 Tacoma 桥风致振动的影响；Feng 和 Tu[6-7]在吊索可松弛条件下用非光滑动力学的方法系统研究了索梁和索板耦合结构在多种支撑条件下的动力学行为．针对不同的数学模型，研究者用解析或数值方法进行了不同侧面的研究[8-10]．但上述工作的不足之处是研究的模型没有同时考虑主缆曲率和吊索松弛对系统的影

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响. 2009 年黄坤等[11]建立了一组新的能反映吊索松弛和主览曲率对系统影响的非线性偏微分方程组，并通过 Galerkin 方法和多尺度方法得到了系统在吊索未松弛条件下非共振情况的一次近似解析解. 此外，冯维明等[12]用多尺度法研究了斜拉桥形式的梁索耦合结构的非线性振动特性. 赵跃宇等[13-14]对多种索结构进行了较系统的研究. 对于悬索桥形式的梁索耦合结构的风致振动机理，自 Tacoma 桥倒塌后一直是研究热点. 该系统不仅存在混沌现象[15]，而且吊索的松弛对系统的动力学行为也存在显著的影响[2]. 本文将在吊索未松弛情况下讨论系统的动力学行为. 通过研究主缆及梁的振幅变化情况来确定吊索可能松弛的情况，为下一步研究系统在吊索松弛情况下的动力学行为做准备.

1　振动微分方程组的建立及 Galerkin 简化

要对物理系统进行合乎实际的研究，建立能反映物理系统主要特性的数学模型是第一步. 本文在文献[11]建立的数学模型的基础上进行讨论. 该模型反映了主缆曲率和吊索变形对系统动力学行为的影响. 在本文中将忽略吊索松弛对系统的影响. 考虑如图 1 所示的计算模型(加劲梁和主缆两端为铰支). 在连续膜假设(连续膜假设是指把吊索看成一张连续张在主缆和梁之间的连续膜)条件下建立如下能反映主缆曲率对系统动力学行为影响的微分方程组：

$$\begin{cases}\dfrac{\partial^2 w}{\partial t^2}+a\dfrac{\partial^4 w}{\partial x^4}+b\dfrac{\partial^5 w}{\partial^4 x\partial t}+c\dfrac{\partial w}{\partial t}+d_1(x)(w-u)=\bar{f}_1(x,t)\\ \dfrac{\partial^2 u}{\partial t^2}-\tilde{g}\dfrac{\partial^2 u}{\partial x^2}-n(x)\dfrac{\partial u}{\partial x}\dfrac{\partial^2 u}{\partial x^2}-q\left(\dfrac{\partial u}{\partial x}\right)^2-h(x)\dfrac{\partial u}{\partial x}-d_2(x)(w-u)=\bar{f}_2(x,t)\end{cases}\tag{1}$$

方程组中的系数和已知函数 $n(x)$，$h(x)$，$d_1(x)$，$d_2(x)$，$\bar{f}_1(x,t)$，$\bar{f}_2(x,t)$见文献[11]. 求解的边界条件为

$$w(0,t)=w(l,t)=\frac{\partial^2 w(0,t)}{\partial x^2}=\frac{\partial^2 w(l,t)}{\partial x^2}=0,\ u(0,t)=u(l,t)=0\tag{2}$$

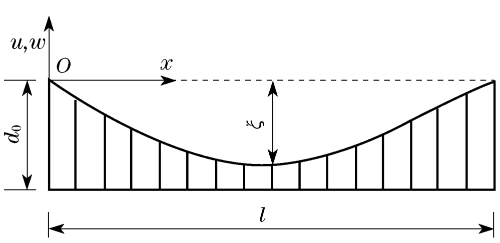

图 1　计算简图

Fig.1　Model of the construct

量纲一方程组(1)后，设 $w=\sum_{i=1}^{\infty}w_i(t)\sin i\pi x$，$u=\sum_{i=1}^{\infty}u_i(t)\sin i\pi x$，仅取其第 1 项代入方程组(1)，之后两边乘以 $\sin \pi x$，并在[0，1]上积分(Galerkin 法一次截断)，可把非线性偏微分方程组(1)化为如下时域上的非线性常微分方程组：

$$\begin{cases}\ddot{w}_1+\alpha\dot{w}_1+\beta w_1-d_{11}u_1=\hat{f}_1\cos\omega_1 t\\ \ddot{u}_1+\gamma u_1+\lambda u_1^2-d_{22}w_1=\hat{f}_2\cos\omega_2 t\end{cases}\tag{3}$$

式中：w_1，u_1 分别为梁及主缆在 1 阶模态振动时的振幅. 变量上的点号表示对时间求导. 求解的初值条件为

$$w_1(0)=w_0,\ u_1(0)=u_0,$$
$$\dot{w}_1(0)=\dot{w}_0,\ \dot{u}_1(0)=\dot{u}_0\tag{4}$$

方程组(3)中的变量和系数 α，β，γ，λ，d_{11}，d_{22}，$\hat{f}_1$，$\hat{f}_2$ 均做了量纲一处理，具体含义见文献[11]. 下文主要讨论非自治常微分方程组(3).

2　多尺度法求解

为了考察作用在梁上的载荷引起共振时的响应，对参数进行重新标度，使非线性项阻尼及作用在梁上的外激励出现在同一个摄动方程中. 令

$$\alpha=\varepsilon\alpha_1,\ \hat{f}_1=\varepsilon^2 f_1,\ \hat{f}_2=\varepsilon^2 f_2\tag{5}$$

把式(5)带入式(3)并省去变量的下标，得

$$\begin{cases}\ddot{w}+\varepsilon\alpha_1\dot{w}+\beta w-d_{11}u=\varepsilon^2 f_1\cos\omega_1 t\\ \ddot{u}+\gamma u+\lambda u^2-d_{22}w=\varepsilon^2 f_2\cos\omega_2 t\end{cases}\tag{6}$$

ε 为量纲一化后振幅的量级. 设式(6)的解为

$$\begin{cases}w=\varepsilon w_1(T_0,T_1)+\varepsilon^2 w_2(T_0,T_1)\\ u=\varepsilon u_1(T_0,T_1)+\varepsilon^2 u_2(T_0,T_1)\end{cases}\tag{7}$$

其中 $T_0=t$，$T_1=\varepsilon t$. 在此有

$$\frac{\mathrm{d}}{\mathrm{d}t}=D_0+\varepsilon D_1,\ \frac{\mathrm{d}^2}{\mathrm{d}t^2}=D_0^2+2\varepsilon D_0 D_1+\varepsilon^2 D_1^2\tag{8}$$

其中 $D_i=\dfrac{\partial}{\partial T_i}$，$D_i^2=\dfrac{\partial^2}{\partial T_i^2}$. 把式(7)、(8)带入式(6)，并令 $\varepsilon,\varepsilon^2$ 的系数相等得

$$\varepsilon:\begin{cases}\dfrac{\partial^2 w_1}{\partial T_0^2}+\beta w_1-d_{11}u_1=0\\ \dfrac{\partial^2 u_1}{\partial T_0^2}+\gamma u_1-d_{22}w_1=0\end{cases}\tag{9}$$

$$\varepsilon^2:\begin{cases}\dfrac{\partial^2 w_2}{\partial T_0^2}+\beta w_2-d_{11}u_2=-2\dfrac{\partial^2 w_1}{\partial T_0\partial T_1}-\\ \qquad\alpha_1\dfrac{\partial w_1}{\partial T_0}+f_1\cos\omega_1 t\\ \dfrac{\partial^2 u_2}{\partial T_0^2}+\gamma u_2-d_{22}w_2=-2\dfrac{\partial^2 u_1}{\partial T_0\partial T_1}-\\ \qquad\lambda u_1^2+f_2\cos\omega_2 T_0\end{cases}\tag{10}$$

设方程组(9)的解为

$$\begin{cases}w_1(x,T_0)=A_1(T_1)\exp(\mathrm{i}\omega_{11}T_0)+\\ \qquad B_1(T_1)\exp(\mathrm{i}\omega_{12}T_0)+\mathrm{cc}\\ u_1(x,T_0)=\mu_1 A_1(T_1)\exp(\mathrm{i}\omega_{11}T_0)+\\ \qquad\mu_2 B_1(T_1)\exp(\mathrm{i}\omega_{12}T_0)+\mathrm{cc}\end{cases}\tag{11}$$

其中 cc 为前面2项的复共轭项，$A_j,B_j,(j=1,2)$为 T_1 的任意函数. 由线性振动理论有

$$\omega_{1j}^2=\frac{1}{2}[(\beta+\gamma)\pm\sqrt{(\beta-\gamma)^2+4d_{11}d_{22}}],$$
$$\mu_j=(\beta-\omega_{1j}^2)d_{11}^{-1}\tag{12}$$

其中 $j=1,2$. 把式(11)带入式(10)得

$$\frac{\partial^2 w_2}{\partial T_0^2}+\beta w_2-d_{11}u_2=-2\left[\mathrm{i}\omega_{11}\exp(\mathrm{i}\omega_{11}T_0)\frac{\partial A_1}{\partial T_1}+\mathrm{i}\omega_{12}\exp(\mathrm{i}\omega_{12}T_0)\frac{\partial B_1}{\partial T_1}\right]-\alpha_1[\mathrm{i}\omega_{11}\exp(\mathrm{i}\omega_{11}T_0)A_1+\mathrm{i}\omega_{12}\exp(\mathrm{i}\omega_{12}T_0)B_1]+\frac{1}{2}f_1\exp(\mathrm{i}\omega_1 T_0)+\mathrm{cc}\tag{13}$$

$$\frac{\partial^2 u_2}{\partial T_0^2}+\gamma u_2-d_{22}w_2=-2\left[\mathrm{i}\omega_{11}\exp(\mathrm{i}\omega_{11}T_0)\mu_1\cdot\frac{\partial A_1}{\partial T_1}+\mathrm{i}\omega_{12}\exp(\mathrm{i}\omega_{12}T_0)\mu_2\frac{\partial B_1}{\partial T_1}\right]-\lambda\{\mu_1^2A_1^2\cdot\exp(\mathrm{i}2\omega_{11}T_0)+\mu_2^2B_1^2\exp(\mathrm{i}2\omega_{12}T_0)+2\mu_1\mu_2A_1B_1\exp[\mathrm{i}(\omega_{11}+\omega_{12})T_0]+2\mu_1\mu_2A_1\bar{B}_1\exp[\mathrm{i}(\omega_{11}-\omega_{12})T_0]+\mu_1^2A_1A_1+\mu_2^2B_1\bar{B}_1\}+\frac{1}{2}f_2\exp(\mathrm{i}\omega_2T_0)+\mathrm{cc}\tag{14}$$

从式(13)，(14)可以看出，系统在振动时除了 $\omega_1\approx\omega_{12}$，$\omega_1\approx\omega_{11}$，$\omega_2\approx\omega_{12}$，$\omega_2\approx\omega_{11}$ 等4种主共振，还存在 $\omega_{11}\approx2\omega_{12}$ 内共振情况. 在工程中，作用在梁上的荷载远大于作用在主缆上的荷载. 故可仅考虑作用在梁上的荷载引起共振的情况，此时有 $\omega_1\approx\omega_{12}$，$\omega_1\approx\omega_{11}$ 和 $\omega_{11}\approx2\omega_{12}$ 等3种情况.

2.1 荷载激起低频模态的情况($\omega_1\approx\omega_{12}$，$\omega_{11}\approx2\omega_{12}$)

2.1.1 发生主共振时

此时令 $\omega_1=\omega_{12}+\varepsilon\sigma_1$，在无内共振时，为了确定式(13)、(14)的可解条件，设式(13)、(14)有如下形式的特解[16]：

$$\begin{cases}w_2=A_{21}\exp(\mathrm{i}\omega_{11}T_0)+B_{21}\exp(\mathrm{i}\omega_{12}T_0)\\ u_2=A_{22}\exp(\mathrm{i}\omega_{11}T_0)+B_{22}\exp(\mathrm{i}\omega_{12}T_0)\end{cases}\tag{15}$$

把式(15)带入式(13)、(14)得

$$\begin{cases}[(\beta-\omega_{11}^2)A_{21}-d_{11}A_{22}]\exp(\mathrm{i}\omega_{11}T_0)+\\ \quad[(\beta-\omega_{12}^2)B_{21}-d_{11}B_{22}]\exp(\mathrm{i}\omega_{12}T_0)=\\ \quad-\left(2\mathrm{i}\omega_{11}\dfrac{\partial A_1}{\partial T_1}+\mathrm{i}\alpha_1\omega_{11}A_1\right)\exp\left(\mathrm{i}\omega_{11}T_0\right)-\\ \quad\left(2\mathrm{i}\omega_{12}\dfrac{\partial B_1}{\partial T_1}+\mathrm{i}\alpha_1\omega_{12}B_1\right)\exp(\mathrm{i}\omega_{12}T_0)+\\ \quad\dfrac{1}{2}f_1\exp(\mathrm{i}\omega_1T_0)+\mathrm{cc}\\ [(\gamma-\omega_{11}^2)A_{22}-d_{22}A_{21}]\exp(\mathrm{i}\omega_{11}T_0)+\\ \quad[(\gamma-\omega_{12}^2)B_{22}-d_{22}B_{21}]\exp(\mathrm{i}\omega_{12}T_0)=\\ \quad-2\mathrm{i}\omega_{11}\exp(\mathrm{i}\omega_{11}T_0)\mu_1\dfrac{\partial A_1}{\partial T_1}-\\ \quad2\mathrm{i}\omega_{12}\exp(\mathrm{i}\omega_{12}T_0)\mu_2\dfrac{\partial B_1}{\partial T_1}-\\ \quad\lambda\{\mu_1^2A_1^2\exp(\mathrm{i}2\omega_{11}T_0)+\mu_2^2B_1^2\exp(\mathrm{i}2\omega_{12}T_0)+\\ \quad2\mu_1\mu_2A_1B_1\exp[\mathrm{i}(\omega_{11}+\omega_{12})T_0]+\\ \quad2\mu_1\mu_2A_1\bar{B}_1\exp[\mathrm{i}(\omega_{11}-\omega_{12})T_0]+\\ \quad\mu_1^2A_1\bar{A}_1+\mu_2^2B_1\bar{B}_1\}+\dfrac{1}{2}f_2\exp(\mathrm{i}\omega_2T_0)+\mathrm{cc}\end{cases}\tag{16}$$

由消除久期项的条件，从式(16)得可解条件为

$$\begin{cases}(\beta-\omega_{11}^2)A_{21}-d_{11}A_{22}=-2\mathrm{i}\omega_{11}\dfrac{\partial A_1}{\partial T_1}-\mathrm{i}\alpha_1\omega_{11}A_1\\ -d_{22}A_{21}+(\gamma-{\omega_{11}}^2)A_{22}=-2\mathrm{i}\omega_{11}\mu_1\dfrac{\partial A_1}{\partial T_1}\end{cases}\tag{17}$$

$$\begin{cases}(\beta-\omega_{12}^2)B_{21}-d_{11}B_{22}=-2\mathrm{i}\omega_{12}\dfrac{\partial B_1}{\partial T_1}-\\ \qquad\mathrm{i}\alpha_1\omega_{12}B_1+\dfrac{1}{2}f_1\exp(\mathrm{i}\sigma_1T_1)\\ -d_{22}B_{21}+(\gamma-{\omega_{12}}^2)B_{22}=\\ \qquad-2\mathrm{i}\omega_{12}\mu_2\dfrac{\partial B_1}{\partial T_1}\end{cases}\tag{18}$$

由线性代数理论可知,线性非齐次方程组中 $A_{21}, A_{22}, B_{21}, B_{22}$ 有非零解的充分必要条件是系数矩阵与增广矩阵的秩相同.从式(12)中 ω_{11}, ω_{12} 的表达式可知式(17),(18)的系数行列式为零,故 $A_{21}, A_{22}, B_{21}, B_{22}$ 有解的充分必要条件为下列行列式为零.即

$$\begin{vmatrix} -2\mathrm{i}\omega_{11}\dfrac{\partial A_1}{\partial T_1}-\mathrm{i}\alpha_1\omega_{11}A_1 & -d_{11} \\ -2\mathrm{i}\omega_{11}\mu_1\dfrac{\partial A_1}{\partial T_1} & \gamma-\omega_{11}^2 \end{vmatrix}=0 \tag{19}$$

$$\begin{vmatrix} -2\mathrm{i}\omega_{12}\dfrac{\partial B_1}{\partial T_1}-\mathrm{i}\alpha_1\omega_{12}B_1+\dfrac{1}{2}f_1\exp(\mathrm{i}\sigma_1T_1) & -d_{11} \\ -2\mathrm{i}\omega_{12}\mu_2\dfrac{\partial B_1}{\partial T_1} & \gamma-\omega_{12}^2 \end{vmatrix}=0 \tag{20}$$

由式(19)、(20)得

$$\begin{cases} \mathrm{i}\kappa_1\dfrac{\partial A_1}{\partial T_1}+\mathrm{i}\kappa_2A_1=0 \\ \mathrm{i}\kappa_4\dfrac{\partial B_1}{\partial T_1}+\mathrm{i}\kappa_5B_1-\kappa_7f_1\exp(\mathrm{i}\sigma_1T_1)=0 \end{cases} \tag{21}$$

其中,

$$\left.\begin{aligned} \kappa_1&=2\omega_{11}(\gamma-\omega_{11}^2+d_{11}\mu_1) \\ \kappa_2&=\alpha_1\omega_{11}(\gamma-\omega_{11}^2) \\ \kappa_4&=2\omega_{12}(\gamma-\omega_{12}^2+\mu_2d_{11}) \\ \kappa_5&=\alpha_1\omega_{12}(\gamma-\omega_{12}^2) \\ \kappa_7&=\frac{1}{2}(\gamma-\omega_{12}^2) \end{aligned}\right\} \tag{22}$$

引入极坐标变换,令

$$\left.\begin{aligned} A_1&=\frac{1}{2}a_1(T_1)\exp[\mathrm{i}\theta_1(T_1)] \\ B_1&=\frac{1}{2}b_1(T_1)\exp[\mathrm{i}\theta_2(T_1)] \end{aligned}\right\} \tag{23}$$

其中 $a_1, b_1, \theta_1, \theta_2$ 为 T_1 的任意实函数.把式(23)带入(21)分离实部和虚部,并通过代数及三角函数运算可把方程组(21)化为如下微分方程组:

$$\begin{cases} a'_1=-\dfrac{\kappa_2}{\kappa_1}a_1,\ b'_1=-\dfrac{\kappa_5}{\kappa_4}b_1-\dfrac{f_1\kappa_7}{\kappa_4}\sin\varphi \\ a_1\theta'_1=0,\ b_1\varphi'=-\dfrac{f_1\kappa_7}{\kappa_4}\cos\varphi+\sigma_1b_1 \end{cases} \tag{24}$$

其中 $\varphi=\sigma_1T_1+\theta_2$,自治常微分方程组(24)的稳态解($T_1\to\infty$)为 $a'_1=b'_1=\theta'_1=\varphi'=0$ 时系统的平衡点.令式(24)右边为零可得确定微分方程组稳态解的超越方程组,通过求解该方程组得如下的稳态解:

$$a_1=0,\ b_1=f_1|\kappa_7|(\kappa_5^2+\sigma_1^2\kappa_4^2)^{-1/2},$$
$$\theta_1=\mathrm{const},\ \varphi=\arctan\left(\frac{\kappa_5}{\kappa_4\sigma_1}\right)\pm\pi \tag{25}$$

式中 π 前的正负号由 φ 所在的象限决定.把式(25)带入式(23)求得 A_1, B_1 后,把 A_1, B_1 带入式(12)和(6),并注意到 $\omega_1=\omega_{12}+\varepsilon\sigma_1$ 可得系统的一次近似解析解

$$\begin{cases} w(x,t)=\varepsilon b_1\cos(\varphi+\omega_1t)+O(\varepsilon^2) \\ u(x,t)=\varepsilon\mu_2b_1\cos(\varphi+\omega_1t)+O(\varepsilon^2) \end{cases} \tag{26}$$

从式(26)可知,在非内共振情况下,当外激励的频率和系统的低阶频率接近时,系统的解实际上是线性问题的解.系统的振动幅值随激励幅值增加而增加.

2.1.2　同时发生内共振和主共振

令 $\omega_{11}=2\omega_{12}+\varepsilon\sigma_2$, $\omega_1=\omega_{12}+\varepsilon\sigma_1$,有 $2\omega_{12}T_0=\omega_{11}T_0-\sigma_2T_1$, $(\omega_{11}-\omega_{12})T_0=\omega_{12}T_0+\sigma_2T_1$.把上述表达带入式(16),通过和非内共振情况相似的推导可得确定 A_1, B_1 的常微分方程组

$$\begin{cases} \mathrm{i}\kappa_1\dfrac{\mathrm{d}A_1}{\mathrm{d}T_1}+\mathrm{i}\kappa_2A_1+\kappa_3B_1^2\exp(-\mathrm{i}\sigma_2T_1)=0 \\ \mathrm{i}\kappa_4\dfrac{\mathrm{d}B_1}{\mathrm{d}T_1}+\mathrm{i}\kappa_5B_1+\kappa_6A_1\bar{B}_1\exp(\mathrm{i}\sigma_2T_1)- \\ \qquad f_1\kappa_7\exp(\mathrm{i}\sigma_1T_1)=0 \end{cases} \tag{27}$$

引进极式记号

$$\left.\begin{aligned} A_1&=\frac{1}{2}a_1(T_1)\exp[\mathrm{i}\theta_1(T_1)] \\ B_1&=\frac{1}{2}b_1(T_1)\exp[\mathrm{i}\theta_2(T_1)] \end{aligned}\right\} \tag{28}$$

把式(28)带入式(27),分离实部和虚部,通过代数和三角运算得

$$\begin{cases} a'_1=-\dfrac{\kappa_2}{\kappa_1}a_1-\dfrac{\kappa_3}{\kappa_1}b_1^2\sin\varphi_1 \\ b'_1=-\dfrac{\kappa_5}{\kappa_4}b_1-\dfrac{\kappa_6}{\kappa_4}a_1b_1\sin\varphi_1-\dfrac{f_1\kappa_7}{\kappa_4}\sin\varphi_2 \\ a_1\varphi'_1=-\dfrac{\kappa_3}{\kappa_1}b_1^2\cos\varphi_1+a_1(2\varphi'_2+2\sigma_1-\sigma_2) \\ b_1\varphi'_2=\dfrac{\kappa_6}{\kappa_4}a_1b_1\cos\varphi_1-\dfrac{f_1\kappa_7}{\kappa_4}\cos\varphi_2-\sigma_1b_1 \end{cases} \tag{29}$$

其中

$$\left.\begin{aligned} \kappa_3&=d_{11}\lambda\mu_2^2 \\ \kappa_6&=2\lambda d_{11}\mu_1\mu_2 \\ \varphi_1&=2\theta_2-\theta_1-\sigma_2T_1 \\ \varphi_2&=\theta_2-\sigma_1T_1 \end{aligned}\right\} \tag{30}$$

系统的稳态响应对应系统的平衡点.有两种可能的

解,第一种情况 $a_1\neq0$,$b=0$,即非内共振时由式(26)给出的解;第二种 $a_1\neq0$,$b\neq0$ 时的解可以通过令方程组(29)右端为零所得的超越方程组给出.通过对该超越方程组的求解可得

$$\begin{cases} b_1^2=\dfrac{\kappa^{1/2}a_1}{|\kappa_3|}, \\ \varphi_1=\arctan\left(\dfrac{-\kappa_2}{\kappa_1(2\sigma_1-\sigma_2)}\right)\pm\pi \\ \varphi_2=\arctan\left(\dfrac{-\kappa_4\sigma_1+a_1\kappa_6\cos\varphi_1}{-\kappa_5-a_1\kappa_6\sin\varphi_1}\right)\pm\pi \end{cases}\tag{31}$$

其中 π 前的正负号由 φ_1,φ_2 所在象限决定,即由式(29)中 $\sin\varphi_1$,$\cos\varphi_1$,$\sin\varphi_2$,$\cos\varphi_2$ 的取值正负情况决定.此外式(31)中的 a_1 由下面的一元三次方程求得

$$e_1a_1^3+e_2a_1^2+e_3a_1+e_4f_1^2=0\tag{32}$$

其中

$$\begin{gathered} e_1=(\kappa_2\kappa_6)^2+(\kappa_1\kappa_6(2\sigma_1-\sigma_2))^2, \\ e_3=\kappa_5^2\kappa+\kappa_4^2\kappa\sigma_1^2,\quad e_4=\kappa_3\kappa_7^2\sqrt{\kappa}, \\ e_2=2(\kappa_5\kappa_2\kappa_6\sqrt{\kappa}-\kappa_4\sqrt{\kappa}\kappa_1\kappa_6\sigma_1(2\sigma_1-\sigma_2)), \\ \kappa=\kappa_2^2+\kappa_1^2(2\sigma_1-\sigma_2)^2 \end{gathered}\tag{33}$$

由式(32)求出 a_1 后带入式(31)求出 b_1,φ_1,φ_2,进而由式(30)求出 θ_1,θ_2 后带入式(7)和(11),并注意到 $\omega_{11}=2\omega_{12}+\varepsilon\sigma_2$,$\omega_1=\omega_{12}+\varepsilon\sigma_1$ 可得出系统的一次近似解

$$\begin{cases} w(x,t)=\varepsilon a_1\cos(2\varphi_2-\varphi_1+2\omega_1t)+ \\ \qquad\varepsilon b_1\cos(\varphi_2+\omega_1t)+O(\varepsilon^2) \\ u(x,t)=\varepsilon\mu_1a_1\cos(2\varphi_2-\varphi_1+2\omega_1t)+ \\ \qquad\varepsilon\mu_2b_1\cos(\varphi_2+\omega_1t)+O(\varepsilon^2) \end{cases}\tag{34}$$

从式(34)可知,内共振起作用时系统的稳态响应是2个周期振动的叠加.

2.2 当激励激起高频模态时

2.2.1 非内共振情况

此时令 $\omega_1=\omega_{11}+\varepsilon\sigma_3$,并代入式(16),和低频共振情况相似,可得 A_1,B_1 满足的方程为

$$\begin{cases} \mathrm{i}\kappa_1\dfrac{\partial A_1}{\partial T_1}+\mathrm{i}\kappa_2A_1-f_1\kappa_8\exp(\mathrm{i}\sigma_3T_1)=0 \\ \mathrm{i}\kappa_4\dfrac{\partial B_1}{\partial T_1}+\mathrm{i}\kappa_5B_1=0 \end{cases}\tag{35}$$

其中 $\kappa_8=\dfrac{1}{2}(\gamma-\omega_{11}^2)$,把未知量记为极式

$$A_1=\frac{1}{2}a_2(T_1)\exp(\mathrm{i}\theta_3(T_1))$$

$$B_1=\frac{1}{2}b_2(T_1)\exp(\mathrm{i}\theta_4(T_1))$$

代人式(35)分离实部和虚部,并经过代数和三角运算后得稳态解满足的方程组为

$$\begin{cases} b'_2=-\dfrac{\kappa_5}{\kappa_4}b_1 \\ a'_2=-\dfrac{\kappa_2}{\kappa_1}a_2-\dfrac{f_1\kappa_8}{\kappa_1}\sin\varphi_3 \\ \theta'_4=0 \\ a_2\varphi'_3=-\dfrac{f_1\kappa_8}{\kappa_1}\cos\varphi_3+\sigma_3 \end{cases}\tag{36}$$

其中 $\varphi_3=\theta_3-\sigma_3T_1$.令式(36)的右端为零,通过求解所得的超越式方程组可得系统的稳态解如下:

$$\left.\begin{aligned} a_2&=f_1|\kappa_8|(\kappa_2^2+\sigma_3^2\kappa_1^2)^{-1/2} \\ b_2&=0 \\ \varphi_3&=\arctan\left(-\frac{\kappa_2}{\kappa_1\sigma_3}\right)\pm\pi \\ \theta_4&=\mathrm{const} \end{aligned}\right\}\tag{37}$$

式中 π 前的正负号由 φ 所在的象限决定.注意到 $\omega_1=\omega_{11}+\varepsilon\sigma_3$,可把非共振时的一次近似解析解写为

$$\begin{cases} w(x,t)=\varepsilon a_2\cos(\varphi_3+\omega_1t)+O(\varepsilon^2) \\ u(x,t)=\varepsilon\mu_1a_2\cos(\varphi_3+\omega_1t)+O(\varepsilon^2) \end{cases}\tag{38}$$

2.2.2 同时发生内共振和主共振的情况

此时有 $\omega_1=\omega_{11}+\varepsilon\sigma_3$,$\omega_{11}=2\omega_{12}+\varepsilon\sigma_2$,和前面的讨论类似,将其代入式(16),经运算后可得 A_1,B_1 满足的方程组

$$\begin{cases} \mathrm{i}\kappa_1\dfrac{\mathrm{d}A_1}{\mathrm{d}T_1}+\mathrm{i}\kappa_2A_1+\kappa_3B_1^2\exp(-\mathrm{i}\sigma_2T_1)- \\ \qquad f_1\kappa_8\exp(\mathrm{i}\sigma_3T_1)=0 \\ \mathrm{i}\kappa_4\dfrac{\mathrm{d}B_1}{\mathrm{d}T_1}+\mathrm{i}\kappa_5B_1+\kappa_6A_1\bar{B}_1\exp(\mathrm{i}\sigma_2T_1)=0 \end{cases}\tag{39}$$

引进极式记号,$A_1=\dfrac{1}{2}a_2(T_1)\exp(\mathrm{i}\theta_3(T_1))$,$B_1=\dfrac{1}{2}b_2(T_1)\exp(\mathrm{i}\theta_4(T_1))$,并代入式(39),分离实部和虚部,整理后得如下自治常微分方程组:

$$\begin{cases} a'_2=-\dfrac{\kappa_2}{\kappa_1}a_2-\dfrac{\kappa_3}{\kappa_1}b_2^2\sin\varphi_4-\dfrac{f_1\kappa_8}{\kappa_1}\sin\varphi_3 \\ b'_2=-\dfrac{\kappa_5}{\kappa_4}b_2+\dfrac{\kappa_6}{\kappa_4}a_2b_2\sin\varphi_4 \\ a_2\varphi'_3=\dfrac{\kappa_3}{\kappa_1}b_2^2\cos\varphi_4-\dfrac{f_1\kappa_8}{\kappa_1}\cos\varphi_3-\sigma_3a_2 \\ b_2\varphi'_4=\dfrac{2\kappa_6}{\kappa_4}a_2b_2\cos\varphi_4-(\varphi'_3+\sigma_2+\sigma_3)b_2 \end{cases}\tag{40}$$

其中的系数如下:

$$\left.\begin{aligned}\kappa_8 &= \frac{1}{2}(\gamma - \omega_{11}^2)\\ \varphi_3 &= \theta_3 - \sigma_3 T_1\\ \varphi_4 &= 2\theta_4 - \theta_3 - \sigma_2 T_1\end{aligned}\right\} \tag{41}$$

令方程组(40)的右边为零可得稳态解满足的列超越方程组,求解该方程组的稳态解为

$$\left.\begin{aligned}a_2 &= \frac{(4\kappa_5^2 + \kappa_4^2(\sigma_2 + \sigma_3)^2)^{1/2}}{2|\kappa_6|}\\ b_2 &= \left[\frac{-c_2 \pm \sqrt{c_2^2 - 4c_1(c_3 - f_1^2\kappa_8^2)}}{2c_1}\right]^{1/2}\\ \varphi_4 &= \arctan\left(\frac{2\kappa_5}{\kappa_4(\sigma_3 + \sigma_2)}\right) \pm \pi\\ \varphi_3 &= \arctan\left(\frac{-\kappa_2 a_2 + \kappa_3 b_2^2 \sin\varphi_3}{-\kappa_1\sigma_3 a_2 + \kappa_3 b_2^2 \cos\varphi_3}\right) \pm \pi\end{aligned}\right\} \tag{42}$$

式中 π 前的正负号由 φ_3,φ_4 所在的象限决定.其中的系数为

$$c_1 = \kappa_3^2$$

$$c_2 = \frac{-2\kappa_2\kappa_3\kappa_5 - \kappa_1\kappa_3\kappa_4\sigma_3(\sigma_3 + \sigma_2)}{\kappa_6}$$

$$c_3 = \frac{[\kappa_2^2 + \kappa_1^2\sigma_3^2](4\kappa_5^2 + \kappa_4^2(\sigma_2 + \sigma_3)^2)}{4\kappa_6^2}$$

由式(42)得 f_1 为实数的 2 个临界值为

$$f_1 = \xi_1 = \frac{(4c_1c_3 - c_2^2)^{1/2}}{2|\kappa_8|\sqrt{c_1}},\ f_1 = \xi_2 = \frac{\sqrt{c_3}}{|\kappa_8|} \tag{43}$$

由上式可知 $\xi_2 > \xi_1$.注意到 $\omega_1 = \omega_{11} + \varepsilon\sigma_3$,可得系统的一次近似解析解为

$$\left\{\begin{aligned}w(x,t) &= \varepsilon b_2\cos\left[\frac{1}{2}(\varphi_3 + \varphi_4 + \omega_1 t)\right] + \\ &\quad \varepsilon a_2\cos(\varphi_4 + \omega_1 t) + O(\varepsilon^2)\\ u(x,t) &= \varepsilon\mu_2 b_2\cos\left[\frac{1}{2}(\varphi_3 + \varphi_4 + \omega_1 t)\right] + \\ &\quad \varepsilon\mu_1 a_2\cos(\varphi_4 + \omega_1 t) + O(\varepsilon^2)\end{aligned}\right. \tag{44}$$

和地频共振相似,在高频内共振时系统的响应是 2 个周期振动的叠加.

3　解的稳定性分析

上文求得的平衡解的稳定性可通过把系统在平衡点处线性化,通过线性化的 Jacobi 矩阵 $\boldsymbol{J}$ 的特征值来判别平衡解的稳定性.通常情况下,当 $\boldsymbol{J}$ 的特征值的实部全为负时,原方程的稳态解是渐进稳定的;当 $\boldsymbol{J}$ 的特征值有至少一个正实部时,原方程的稳态解是不稳定的;当 $\boldsymbol{J}$ 的特征值有零实部,原方程稳态解的稳定性须进一步讨论.在低频共振时非内共振和内共振的 Jacobi 矩阵分别由方程组(24)、(29)求得.在高频共振时非内共振和内共振的 Jacobi 矩阵分别由方程组(36)、(40)求得.

4　算例及讨论

为了检验解析解的精度并细致讨论系统的动力学特性,现以如下的数据为例进行分析和数值计算:$\alpha = 0.1$,$\beta = 15$,$d_{11} = 9$,$\gamma = 20$,$\omega_2 = 4$,$d_{22} = 11.5$,$\lambda = -1$,$\varepsilon = 0.1$,$\hat{f}_2 = 0.01$.

在激起低阶模态时取 $\omega_1 = 2.63$,激起高阶模态时取 $\omega_1 = 5.3$.相应的各参数如下:$\omega_{11} = 5.29$,$\omega_{12} = 2.65$,$\mu_1 = -1.44$,$\mu_2 = 0.87$,$\kappa_1 = -221.6$,$\kappa_2 = -42$,$\kappa_3 = -6.81$,$\kappa_4 = 110.3$,$\kappa_5 = 34.4$,$\kappa_6 = 22.6$.

4.1　低频共振时

此时有 $\kappa_7 = 6.5$.把上述数据带入式(25)、(31)得如下的以激励幅值为参数的响应幅值曲线(图 2).图中实线为可以实现的稳定解,虚线为不能实现的不稳定解.

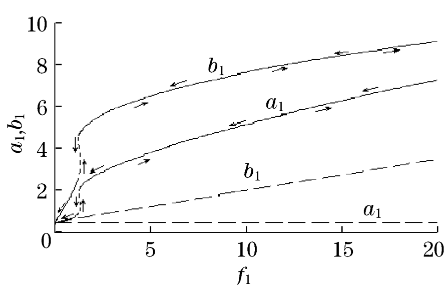

图 2　激发低阶模态的振幅响应

Fig.2　Amplitude response curves of the low-order mode

从图 2 中可以看出,系统一开始振动内共振就起作用,系统的高频振动和低频振动同时被激发.在激励幅值较小时振幅随激励幅值增大而较快地增大,之后增幅逐渐减小.系统的响应幅值在激励幅值较小时就有突然变化的跳跃现象.

本节中比较了在 $\hat{f}_1 = 0.2$ 时 u 数值解和解析解(图 3);并给出了 $\hat{f}_1 = 0.2$ 时 w,u 的时程图(图 4)和 u,$\dot{u}$ 相图(图 5).

从时程图可以看出,主缆的变形幅值和梁的变形幅值相近.这说明当外激励激起系统低阶模态时不能忽略主缆的变形.从解析解和数值解的对比(图 3)可以看出,解析解有很好的精度.由式(34)可知,在内共振起作用时梁和主缆都是 2 个周期振动的叠加,但数值模拟没有明显地表现出该种 2 倍周期运

动.这说明若要较精确地研究系统的分岔行为需要考虑 ε^2 量级的项对系统的影响.

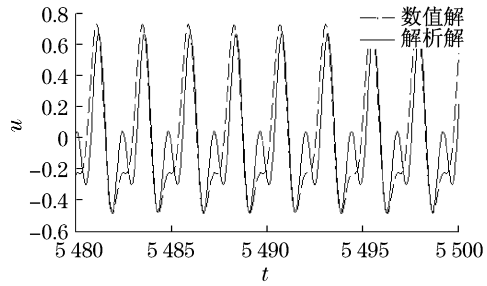

图3 $\hat{f}_1=0.2$ 时 u 的解析解和数值解的对比

Fig.3 Comparison between numerical solution and analytical solution to u with $\hat{f}_1=0.2$

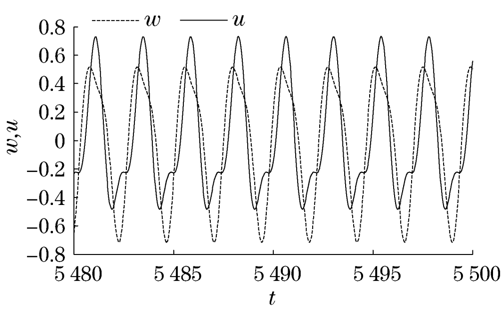

图4 $\hat{f}_1=0.2$ 时 w,u 的时程图

Fig.4 Time histories of w and u with $\hat{f}_1=0.2$

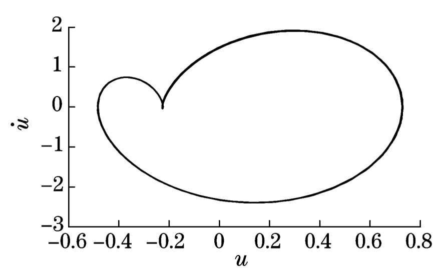

图5 $\hat{f}_1=0.2$ 时 $u,\dot{u}$的相平面图

Fig.5 Phase plane for u and $\dot{u}$ with $\hat{f}_1=0.2$

4.2 高频共振时

此时有 $\kappa_8=-3.99$.把前述数据带入式(37),(42)得如下的以激励幅值 f_1 为参数的 a_2,b_2 响应幅值曲线(图6).图中实线为可以实现的稳定解,虚线为不能实现的不稳定解.

从图6可知,在激励幅值 f_1 的2个分岔值 ξ_1 和 ξ_2 处 a_2 及 b_2 沿不同的曲线变化,系统的振幅发生跳跃现象.比较图2和图6可以发现,尽管荷载激发高阶或低阶模态时系统都会发生跳跃现象,但低频共振时 f_1 的分岔值远远小于高频时的分岔值.这说明系统在低频共振时更容易产生大幅振动.同时,系统存在跳跃现象可以给出 Tacoma 桥在倒塌前竖向振动幅值增大现象[2]的解释.图6还揭示出在高阶共振时,相应于高频 ω_{11} 的振幅 a_2 有饱和现象.此外,从式(43)及 κ_2,κ_5,κ_6 的表达式可知 a_2 和 b_2 发生跳跃的临界值 ξ_2 随阻尼系数 α 及内外共振的协调参数 $|\sigma_i|(i=2,3)$ 增大而增大,随非线性项的系数 λ 的增大而减小.上述结果为数值计算所证实.这说明结构阻尼对抑制梁索耦合结构的振动有十分重要的作用,而增大主缆的曲率则会减小系统产生跳跃现象的临界值.

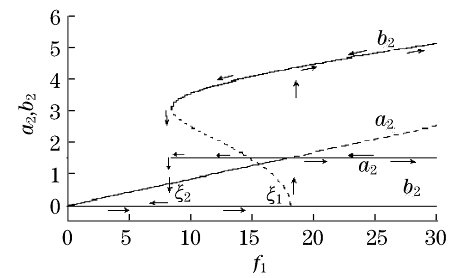

图6 激发高频共振时的振幅响应曲线

Fig.6 Amplitude response curves of the high-order mode

4.2.1 $\hat{f}_1=0.1$ 的时程图和相图

本节中给出了$\hat{f}_1=0.1$ 时 w,u 的时程图(图7)和 $w,\dot{w}$相图(图8).

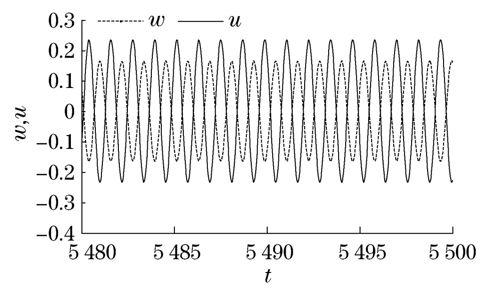

图7 $\hat{f}_1=0.1$ 时 w,u 的时程图

Fig.7 Time histories of w and u with $\hat{f}_1=0.1$

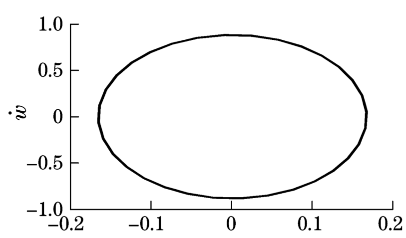

图8 $\hat{f}_1=0.1$ 时 $w,\dot{w}$的相平面图

Fig.8 Phase plane for w and $\dot{w}$ with $\hat{f}_1=0.1$

4.2.2　$\hat{f}_1=0.3$ 的时程图和相图

本节中比较了在$\hat{f}_1=0.3$ 时 u 数值解和解析解(图9);并给出了$\hat{f}_1=0.3$ 时 w,u 的时程图(图10)和 u,$\dot{u}$ 相图(图11).

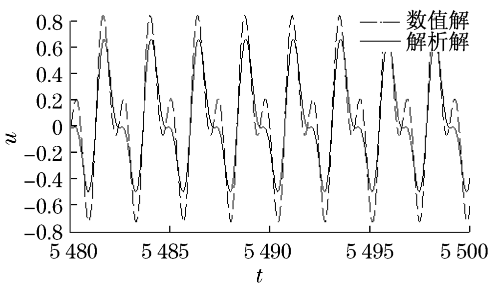

图9　$\hat{f}_1=0.3$ 时 u 的解析解和数值解的对比

Fig.9　Comparison for numerical solution and analytical solution to u with $\hat{f}_1=0.3$

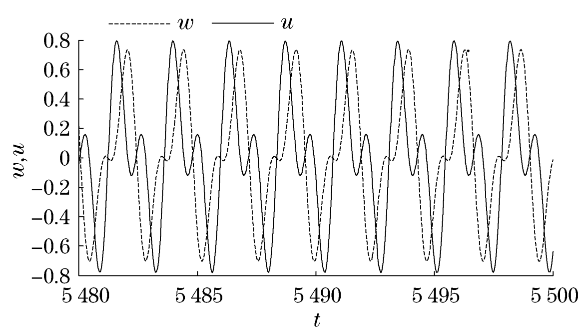

图10　$\hat{f}_1=0.3$ 时 w,u 的时程图

Fig.10　Time histories of w and u with $\hat{f}_1=0.3$

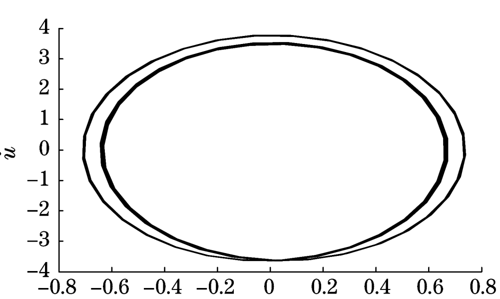

图11　$\hat{f}_1=0.3$ 时 u,$\dot{u}$ 的相平面图

Fig.11　Phase plane for u and $\dot{u}$ with $\hat{f}_1=0.3$

从时程图可以看出,梁的振幅和主缆的振幅相近.在振动过程中有梁向上而索同时向下的运动,此时吊索可能发生松弛.因此,为了能更好地认识梁索耦合结构的动力学特性,有必要考虑吊索松弛对系统的影响.从解析解和相图可以看出,当 f_1 在 ξ_2 处分岔后,系统的响应是2个周期运动的叠加.作用在主缆上的荷载对系统的影响很小.从数值计算和解析解的比较(图9)可知,解析解有较好的精度.

5　结论

从上面对在余弦动荷载作用下悬索桥形式的梁索耦合结构的解析和数值研究可以看出,若系统产生主共振和1∶2内共振,则系统在静平衡位置附近的振动有如下性质:

(1) 作用在梁上的荷载无论是激起低阶共振还是激起高阶共振,梁和主缆的振幅都相近.故在分析梁索耦合系统的动力学行为时不能忽略主缆的变形.

(2) 以作用在梁上的荷载幅值为参数,则系统在低频共振时发生振幅跳跃的分岔值远远小于高频共振时的分岔值.这说明低阶共振更容易使系统产生大幅振动.

(3) 在发生高频共振时,系统出现梁向上而索同时向下的运动.在此情况下系统的吊索可能发生松弛.故有必要在高频共振时研究吊索松弛对系统的动力学影响.

(4) 本文得到的一次近似解析解有良好的精度.

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